Probabilistic System Identification of Two Flexible Joint Models

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The flexibility of welded joints is an important issue in structural analysis and design of car bodies. Two three-dimensional, design-oriented models (uncoupled and coupled) are developed to represent the complaint behavior of multibranch flexible joints. The uncoupled model consists of torsional springs restraining the relative rotation of the joint branches in the three planes, while all branches are assumed to be rigidly connected in translation. Coupling between motions in different planes is neglected. The coupled model accounts for such coupling. A statistical system identification method is proposed for inferring the model parameters from the static response of the structure. The method is demonstrated by applying it to a simple cube frame structure and a car body. Finally, the two models are compared in terms of their ability to predict static response.

Nomenclature

E_{complex} = strain energy of complex model
E_m = error vector
E_{simple} = strain energy of simple model
F = force vector
F_R = reduced force vector
K = stiffness matrix
K_J = joint stiffness matrix
K_{overall} = overall stiffness matrix
K_R = reduced stiffness matrix of structure without the joint
K_U = stiffness matrix of unconstrained joint
k_{ij} = elements of joint stiffness matrix
k_J = joint stiffness parameter vector
L_f = length of force vector
U = displacement vector
U_A = vector of actual displacements
U_E = strain energy stored by the springs
U_m = measured displacement vector
u = rotation vector
V_m = covariance matrix of measurement errors
X_m = matrix formed from the measured displacements
θ_{ij} = rotational degrees of freedom at joint

I. Introduction

THE flexibility characteristics of welded joints are important in the design of automotive structures because, in some cases, they dominate the static and dynamic response.[1] The flexibility of the joints poses a major difficulty in analysis and design because it is not completely understood. Detailed finite element models of the welded joints have been developed,[2] but they cannot be used in design because they are very complex and they involve a high number of degrees of freedom. Simple, design-oriented models of flexible joints do not exist.

Significant improvement in the accuracy of structural analysis and the effectiveness of design optimization is expected when the flexibility of the joints is accounted.[3] Garstecki[4] studied optimal design of joints in linear elastic beams and frames. Ioannidis and Kounadis[5] studied the effect of joint flexibility on the nonlinear buckling load of metal portal frames using springs. Their analysis assumes that the performance of a joint can be simulated by using linear rotational springs. One possible approach is to use test results to identify joint characteristics. For example, Boroski et al.[6] introduced fictitious linear springs to represent the flexibility of joints. Their analysis assumes various spring constants, and then those constants are tuned to make theoretical predictions match dynamic test results. This approach is deterministic and does not provide information on the quality of the estimates.

Baruch[7] and Berman and Nagy[8] developed a system identification procedure to estimate the parameters of a structural system from dynamic test results. They determined the parameters by identifying a set of minimum changes in the original stiffness or mass matrices so that the analytical mode shapes agree with test measurements. As it is often difficult to measure the mode shapes of a vibrating system, Baruch and Khatri[9] introduced methods for identifying the mode shapes from dynamic response measurements. Butkunas et al.[10] identified regions in the models of large automotive structures that should be changed to bring finite element predictions in exact agreement with experimental results. These approaches are deterministic and the model is calibrated so that theoretically predicted natural frequencies and mode shapes exactly match measurements.

A probabilistic approach for system identification for nonlinear systems using a recursive filtering algorithm was proposed by Yun and Shinozuka.[11] Collins et al.[12] developed a statistical identification method that uses measurements of mode shapes and natural frequencies to estimate the parameters of a linear dynamic system. This method essentially consists of repeated application of a sequential linearization technique (see, e.g., Draper and Smith[13] and Bard[14]).

The objectives of this paper are to develop simple finite element models for flexible joints and to demonstrate how probabilistic system identification can be used to identify flexible joint parameters. A weighted regression analysis is applied to experimental measurements or results from very detailed finite element analysis to estimate joint parameters. The approach is demonstrated for a simple model of a flexible joint using computer simulated experimental measurements.

Section II describes two flexible joint models. The uncoupled joint model includes torsional springs restraining the

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relative rotation of the joint branches in the three planes, while all branches are assumed to have the same linear displacements. Coupling between the different planes is neglected. The coupled joint model is defined by its stiffness matrix, and it accounts for the coupling between the motions of the branches of the joint. These two joint models are incorporated in the finite element model of the overall structure, and the stiffness parameters of these models are determined analytically by using experimentally measured displacements.

In Sec. III, the estimation problem is formulated. The estimation problem reduces to the minimization of the residual of the displacements, which is a nonlinear function of the stiffness parameters of the reduced stiffness matrix. Section IV details the solution procedure. The estimation problem is solved by employing two iterative procedures for rapid convergence. In the first stage of the estimation procedure, a fixed point iteration method is used to obtain a first approximation to the joint parameters. Newton's method is employed to refine the approximation. Newton's method requires repeated evaluation of the derivatives of displacements with respect to the joint stiffness parameters. These derivatives are efficiently evaluated using substructuring to condense all of the degrees of freedom that are neither included in the estimation process nor belong to the joint.[15,16] This strategy reduces the cost of sensitivity calculations to a small fraction of the cost of a detailed finite element analysis.

Since experimental measurements were not available, we employ in Sec. V a procedure that contaminates analytically calculated displacements with random noise to simulate displacement measurements in the computer. These values are then used in the statistical system identification procedure to estimate the joint stiffness parameters. Section VI presents the examples used to test the proposed method.

In Sec. VII, we compare the two models. The coupled model is assumed to exactly represent the actual structure. The parameters of both models are estimated using the same measurements that are obtained from the actual structure, and we compare the models in terms of their strain energies. Eigenvalue analysis is used to find the loading case that maximizes the difference in strain energies of the two models. Since the only difference between the two joint models is in accounting for coupling between the motions of the branches, the results of this comparison can be used to assess the importance of the coupling.

II. Joint Models

In this section, we present two models for a flexible joint. The first one, which is called here the uncoupled model, is defined in terms of a small set of parameters, but ignores coupling between the motions of the joint branches. A higher number of parameters is required to define a second model, called here the coupled model, that accounts for coupling between the motions of the joint branches.

The development of the models is based on the following assumptions: 1) the joint is rigid in translation, 2) the joint behavior is linear, and 3) the joint branches rotate about the same point, which will be called geometrical center.

A. Uncoupled Joint Model

In this study, a joint connecting n space frame elements is simulated by $3\binom{n}{2}$ linear torsional springs. Each spring constrains the relative rotation of two elements in a plane. The torsional spring constants are represented by three subscripts. Each torsional spring connects a pair of elements, which are represented by the second and third subscripts, and it constrains their relative rotation in the plane represented by the first subscript. Figure 1 depicts a model of a joint with three branches in X plane. Here, for example, spring k_{x12} connects the pair of elements 1 and 2 in the X plane and constrains the relative rotation between these two elements in the X plane. θ_{x1} refers to the roation of member 1 in the X plane.

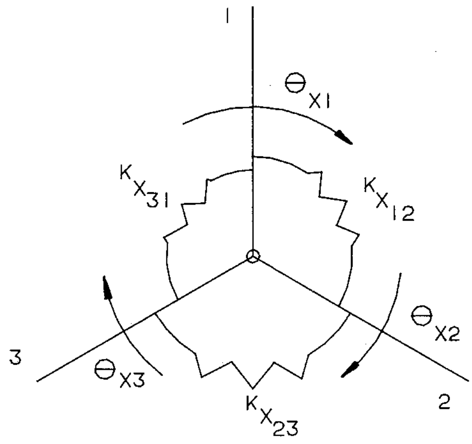

Fig. 1 Simulation of three-branch flexible joint in X plane (uncoupled model).

The stiffness matrix of the uncoupled joint model is such that the nondiagonal terms, which correspond to rotations at different planes, are zero. The nonzero terms represent the coupling motions in the same plane.

There are cases for which the connection between certain degrees of freedom forming the joint is practically rigid. In such cases, the joint stiffness matrix is obtained by ignoring the springs connecting these degrees of freedom and deleting the rows and columns corresponding to connections that are practically rigid.

B. Coupled Joint Model

Consider an n-branch joint. The joint is represented by an element that consists of n nodes. The joint is connected to the rest of the structure through these nodes. According to assumption (3) of Sec. II, all nodes are located at the same position, which is the geometrical center of the joint. Each node can move relatively to the others. In this case, we can model the joint by an element with $6n$ degrees of freedom. According to the assumption that the joint is rigid in translation, the linear displacements of the n nodes are all equal. Thus, the size of the stiffness matrix of the joint becomes $(3n + 3) \times (3n + 3)$ after condensing the degrees of freedom corresponding to the translational displacements of $(n - 1)$ nodes.

The elements of the reduced stiffness matrix of the joint constrained against rigid-body motion are sufficient to determine the stiffness matrix of the unconstrained joint. Consider that the three rotations and three translations of a node of the joint element are constrained to be zero. Then the number of degrees of freedom of the constrained joint element is $(3n - 3)$. As a result, the number of independent parameters that are necessary to determine the full stiffness matrix of the element representing an n-branch joint is $3(n - 1)(3n - 2)/2$. Thus, we need six independent parameters to represent a 2-branch joint by the coupled model. For a 3-branch joint, the number of parameters is 21.

The difference between the stiffness matrices of the uncoupled and the coupled model is that the latter is fully populated. Thus, for the case of the coupled model, a moment that acts in a particular plane will induce rotations of the same branch or the other branches in the other planes. However, if the same moment is applied to the uncoupled model, it will not induce rotations in the other planes.

III. Formulation of the Estimation Problem

In this section, we formulate the problem of identifying the values of the stiffness parameters of the joint by bringing it into the form of a nonlinear estimation model.

Consider a finite element model of the vehicle structure or a part thereof. The matrix form of the equation describing the response of the structure under static loads is

$$K(\boldsymbol{k}_J)\boldsymbol{U} = \boldsymbol{F} \tag{1}$$

Consider a set of measured displacements U_m, and let $\boldsymbol{E}_m$ denote the error between analytical predictions and measured displacements. The following equation relates the measurements with the stiffness parameters

$$\boldsymbol{U}_m = K(\boldsymbol{k}_J)^{-1}\boldsymbol{F} + \boldsymbol{E}_m \tag{2}$$

Observing that the inverse stiffness matrix is a known function of the stiffness parameters, Eq. (2) can be recast into the form

$$\boldsymbol{U}_m = \boldsymbol{U}_A(\boldsymbol{k}_J) + \boldsymbol{E}_m \tag{3}$$

where, $\boldsymbol{U}_A(\boldsymbol{k}_J) = K(\boldsymbol{k}_J)^{-1}F$ is a vector of known functions of the parameters in $\boldsymbol{k}_J$ that are, in general, nonlinear.

The objective when solving a nonlinear estimation model of the form of Eq. (3) is to estimate the vector of parameters $\boldsymbol{k}_J$ from the observation vector $\boldsymbol{U}_m$ and the statistics of the vector of measurement errors $\boldsymbol{E}_m$. The latter is assumed to be a Gaussian vector with zero mean, so that its covariance matrix V_m is sufficient to describe its statistics.

The discrepancy between experimental and analytically predicted displacements is due to 1) random errors in measuring deflections, and 2) systematic errors resulting from approximations and idealizations on which the finite element model is based.

The effect of random measurement error can be averaged out by taking a large number of displacement measurements. Unfortunately, we cannot do the same with the systematic modeling error. In this study, we neglected this systematic error when estimating the model parameters. This simplification induces bias in the estimates. The effect of this simplification is investigated in this paper.

Usually, we measure only a small subset of displacements in the structure. The degrees of freedom that are not measured are eliminated by static condensation and the stiffness matrix in Eq. (2) is replaced by the reduced stiffness matrix. This results in substantial computational savings.[15] The model represented by Eq. (3) is solved by the procedure described in the following section in order to estimate the model parameters.

IV. Solution to the Estimation Problem

Most popular estimation methods find the values of the parameters by minimizing a norm of the vector of residuals. The norm used in this paper is the inner product of the vector of residuals weighted by the inverse of the covariance matrix of displacement measurements. This leads to the weighted least square estimation.[13] The estimation problem reduces to the minimization of a nonlinear function of the stiffness parameters.

This problem is solved by two iterative procedures for fast convergence, as shown in Fig. 2. The solution starts with a fixed point iteration and concludes with Newton's method. The estimation method is applicable to both simple and complex joint models.

A. Fixed-Point Iterative Scheme

The flexible joint model is formed by adding the joint stiffness matrix to the reduced (statically condensed) stiffness matrix K_R

$$[K_R + K_J]\boldsymbol{U}_A = \boldsymbol{F}_R \tag{4}$$

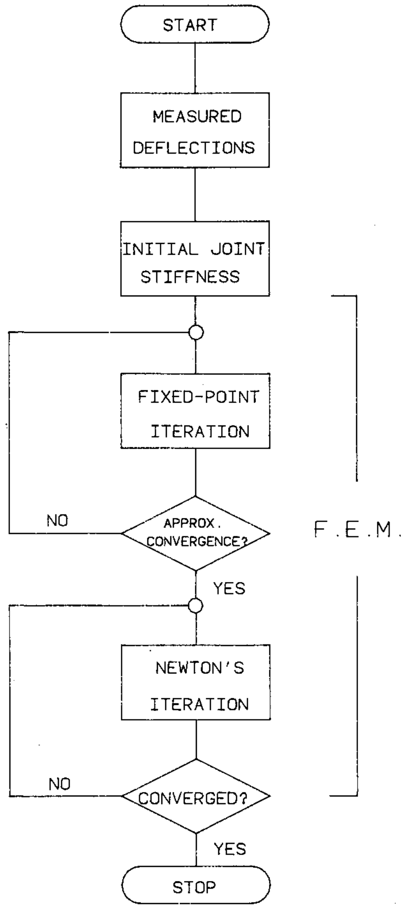

Fig. 2 System identification solution procedures.

Introducing the error in measured displacements,

$$\boldsymbol{U}_A = \boldsymbol{U}_m - \boldsymbol{E}_m \tag{5}$$

Substituting back into Eq. (4) yields

$$X_m\boldsymbol{k}_J = \boldsymbol{F}_R - \boldsymbol{F}_m + K_{\text{overall}}\boldsymbol{E}_m \tag{6}$$

where

$$\boldsymbol{F}_m = K_R\boldsymbol{U}_m, \qquad K_{\text{overall}} = K_R + K_J$$

The entries of X_m are linear combinations of the measured displacements and $\boldsymbol{k}_J$ is a vector containing the stiffness parameters of the joint model.

Measured displacements are subdivided into two classes: 1) measurements at the joint boundary degrees of freedom, and 2) measurements at degrees of freedom away from the joint boundary.

Equation (6) is of the form

$$\boldsymbol{Y} = X\boldsymbol{b} + \boldsymbol{E} \tag{7}$$

where

$$\boldsymbol{Y} = \boldsymbol{F}_R - \boldsymbol{F}_m$$
$$X = X_m$$
$$\boldsymbol{b} = \boldsymbol{k}_J$$
$$\boldsymbol{E} = K_{\text{overall}}\boldsymbol{E}_m$$

We start with an initial guess of the joint parameters in $\boldsymbol{k}_J$ and use it to calculate the matrix K_{overall} on the right side of

Eq. (6). Then, we update k_J by obtaining a weighted residual regression solution for Eq. (6). The solution is given by the following formula,

$$b = [X^T V_e^{-1} X]^{-1} X^T V_e^{-1} Y \tag{8}$$

where

$$V_e = K_{\text{overall}} V_m K_{\text{overall}}^T \tag{9}$$

The covariance matrix of the parameter estimates is given by

$$V_b = (X^T V_e^{-1} X)^{-1} \tag{10}$$

B. Newton's Method

The nonlinear model [Eq. (4)] can be linearized about a set of nominal values of the stiffness parameters,

$$U_m = [K_R + K_J]^{-1}_{\text{nominal}} F_R + \left[\frac{\partial U}{\partial k_J}\right]_{\text{nominal}} \Delta k_J + E_m \tag{11}$$

Newton's iteration uses Eqs. (8) and (9), by substituting $[\partial U/\partial k_J]_{\text{nominal}}$ for X and V_m for V_e, to estimate the solution and the standard deviation of the parameter estimates. These standard deviations indicate the effect of measurement error on the estimates. Therefore, they can be used to decide if we have taken enough measurements so that the effect of this error has been averaged out. The fixed-point iteration and Newton's method are presented in more detail in Ref. 16.

V. Simulation of Displacement Measurements

An actual joint is not simply a set of beams whose ends coincide. It is a geometrically complex assembly of overlapping metal sheets joined together. For each branch, we should measure the rotations of a number of nodes in the vicinity of the branch end. We can consider their average as measured rotations of the node at the branch end in the joint model.

The rotations of a joint branch are superpositions of rotations due to movement of the joint as a rigid body and due to relative motions between the joint members. Only the relative motions are useful in estimating the joint parameters. Unfortunately, the former are considerably larger than the latter. Therefore, it is difficult to extract the relative rotations of the joint branches from their measured rotations. This results in a very high sensitivity of the estimated joint parameters to the error in measurements.

We can significantly reduce this undesirable effect of measurement errors as follows. The rotation of a reference point in the vicinity of a joint is measured first. Then, we measure the relative rotations of all other points on the joint branches with respect to this reference point. Finally, the rotation of each point is obtained by adding the relative rotations to the rotation of the reference point. In that case, the relative branch rotations, which are entries of matrix X_m, are not contaminated with rigid-body rotations. Therefore, the estimated parameters are significantly more accurate.

The error in the rotations consists of three parts: 1) error due to model simplification, 2) measurement error, and 3) error due to the representation of rotations by an average rotation of points near the joint.

Only the measurement error can be averaged out by taking many loading cases. Errors in measurements are assumed to be independent. Therefore, their covariance matrix V_m is diagonal with the variances of each measurement error in the diagonal. We use random noise to simulate the measurement error. The simulated measurements are then used in the statistical system identification procedure. The results below may be expected to be better than those obtained from actual experiments because errors 1 and 2 are not simulated.

The procedure used to simulate measurements starts with a finite element analysis to yield the exact values of displacements. These are contaminated by random noise to generate the simulated measured displacements. Using a set of initially guessed values for the joint stiffness parameters, we obtain the corresponding displacements that are compared with the simulated measured displacements and the estimation process is carried out. The new values of joint parameters are substituted for the previously used values and the process is repeated to convergence.

VI. Examples

We demonstrate the proposed method by identifying joint parameters of two structures, a cube frame assembly and a car body. Only the parameters of the uncoupled model are estimated.

For all of the examples considered in the following sections, the loads consist of forces and/or moments that are applied to the nodes corresponding to the ends of the beams that are attached to the joint. Displacements and rotations are measured at the nodes corresponding to the joint. For some cases, the displacements and rotations of nodes that are away from the joint are also measured.

A. Illustrative Cube Frame Example

A cube frame with a flexible joint at node 7 (see Fig. 3) is used as the first example. Node 1 is fixed and the frame is modeled by space frame elements having 6 degrees of freedom (DOF) per node. Geometry and material properties are given in Table 1. The spring rates in the simple joint model are all taken to be equal to a single number k equal to 1.0×10^4 N·m. The standard deviations of the errors are 5% of the respective translations or rotations.

Figure 4 demonstrates that the use of fixed-point iteration is beneficial as compared to using only Newton's method in the identification process. For this example, we perform three iterations of the fixed-point iteration scheme and then use Newton's iteration to converge to the final solution. It is found that we need only four iterations for convergence. However, if only Newton's method is used, nine iterations are required for convergence.

The stiffness parameters of the joint model can also be estimated by using a number of measurements equal to the number of parameters and by solving the system of equations relating the displacements of the parameters. Table 2 compares this approach, which is labeled the deterministic approach, to probabilistic system identification. The identified values are presented in this table. Clearly, probabilistic identification is more effective than the deterministic approach because the number of equations that are used to estimate the unknown parameters is larger than the number of these

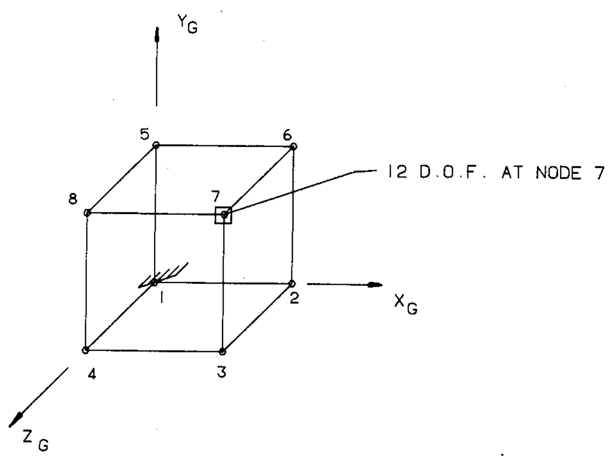

Fig. 3 Cube example with flexible joint at node 7.

Table 1 Frame element properties for cube model

Property	Value
Young's modulus, E	200 GPa
Shear modulus, G	80 GPa
Torsional constant, J	0.4E-08 m^4
Cross-section area, A	4.0E-04 m^2
Moment of inertia, I_{yy}	1.333E-08 m^4
Moment of inertia, I_{zz}	1.333E-08 m^4
Length of element, L	0.2 m
Product of inertia, I_{yz}	0.0 m^4

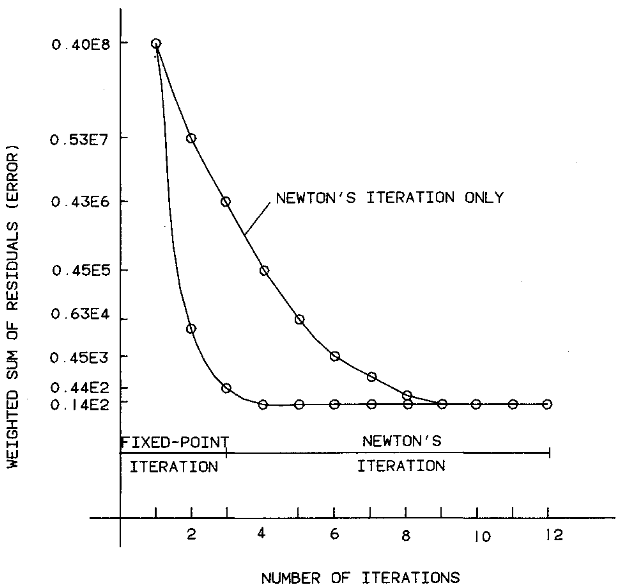

Fig. 4 Comparison of convergence of combined fixed-point iteration and Newton's iteration vs Newton's iteration only (cube example).

Table 2 Probabilistic vs deterministic identification (cube example)

Joint stiffness parameter	Deterministic approach	Probabilistic approach	
	Value, $N{\cdot}m \times 10^4$	Value, $N{\cdot}m \times 10^4$	Standard deviation (percent of estimated value)
k_{x12}	1.2562	1.0097	0.67
k_{y12}	0.9663	1.0112	0.60
k_{z12}	0.9775	1.0147	0.48
k_{x23}	0.9726	0.9982	1.22
k_{y23}	1.3021	1.0076	0.99
k_{z23}	1.1297	0.9978	0.61
k_{x31}	0.9514	0.9937	1.02
k_{y31}	0.9370	0.9945	0.41
k_{z31}	0.9662	1.0184	0.79

parameters. As a result, the effect of the measurement error on the parameter estimates is averaged out, and, therefore, these estimates are more accurate than those obtained deterministically.

Errors resulting from approximations and idealizations on which the finite element model is based induce systematic bias in the estimated stiffness of the joint parameters. This systematic bias is now simulated in the estimation process through the forces by increasing or decreasing the applied forces by a scale factor. This also can simulate bias in the stiffness matrix of the flexible joint and the structure.

Figure 5 gives the block diagram of the biased process of estimation. Figure 6 shows the estimate of the parameter k_{z31} (cube model) as a function of the number of loading cases with confidence intervals for both the biased and unbiased cases. The bias used is 0.9 for both the cases when loads are applied at the joint and when loads are applied away from the joint. It is observed that the standard deviation of the estimates obtained by using measurements from 10 loading cases is roughly 5% of the estimates. As expected, the bias causes the error in the estimates not to converge to zero as the number of loading cases tends to infinity.

B. Illustrative Car Model Example

A car body with one flexible joint shown in Fig. 7 is analyzed. The car is modeled by frame finite elements with 6 DOF per node. All of the beams in the model are identical and their properties are given in Table 3. The model has 63 nodes and 115 elements with 384 DOF, including the 12 DOF for the flexible joint. For the uncoupled model, the stiffness parameters are all equal to 1.15×10^5 N·m.

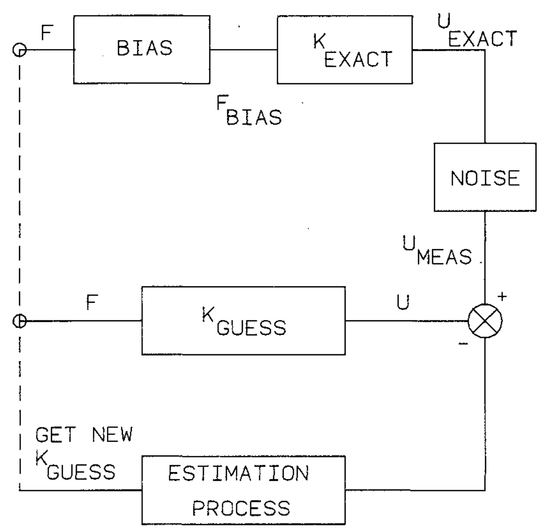

Fig. 5 Biased estimation process.

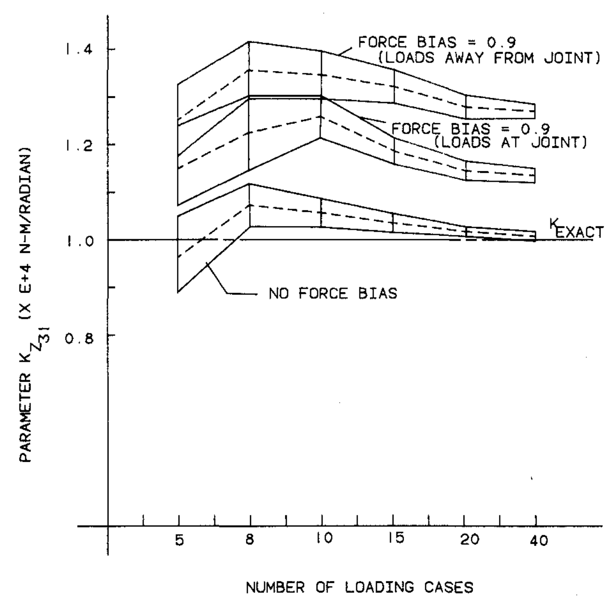

Fig. 6 Effect of force bias on estimation of parameter k_{z31} (cube example).

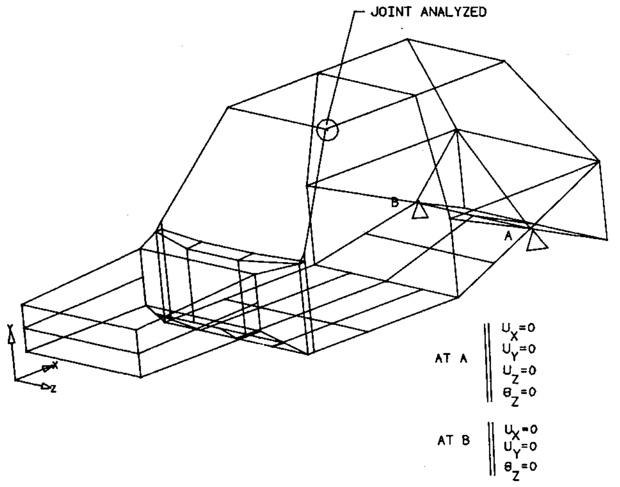

Fig. 7 Space frame stick model of car.

Table 3 Frame element properties for car model

Property	Value
Young's modulus, E	206.85 GPa
Shear modulus, G	82.7 GPa
Torsional constant, J	35.4E-08 m^4
Cross-section area, A	2.99E-04 m^2
Moment of inertia, I_{yy}	16.65E-08 m^4
Moment of inertia, I_{zz}	77.42E-08 m^4
Product of inertia, I_{yz}	0

Table 4 shows the estimated values and confidence intervals of the estimated parameters, based on five loading cases, for two cases. In the first case, the parameters are estimated by measuring rotations only at the joint boundaries, whereas in the second case, additional measurements at points away from the joint are also used. These measurements correspond to degrees of freedom at the ends of the members forming the joint. It is observed that the estimates are better for the second case.

The number of loading cases was gradually increased with 5, 8, 10, 15, and 20 loading cases used. Table 5 gives the standard deviations of the estimated stiffness parameters. It is observed that, as the number of loading cases increases, the confidence interval decreases, which indicates that the quality of estimates improves. Five loading cases were sufficient to give reasonable estimates with a standard deviation, which is 5% of the actual value.

Table 5 Effect of number of load cases on standard deviation of parameter estimates (car example)

Joint stiffness parameter	Standard deviation (percentage of actual value) 5 cases	8 cases	10 cases	15 cases	20 cases
k_{x12}	2.09	1.61	1.28	0.61	0.47
k_{y12}	2.42	1.94	1.58	0.88	0.56
k_{z12}	3.82	2.97	2.17	1.13	0.68
k_{x23}	3.33	2.33	1.85	1.08	0.64
k_{y23}	2.81	1.98	1.43	0.92	0.55
k_{z23}	2.07	1.57	1.23	0.54	0.41
k_{x31}	5.30	3.79	2.33	2.00	1.23
k_{y31}	1.30	0.88	0.63	0.51	0.37
k_{z31}	3.94	3.12	1.79	1.45	1.07

VII. Comparison of the Uncoupled and Coupled Models

The coupled model is more realistic because it accounts for the coupling between the motions of the joint branches. On the other hand, it is more complicated than the uncoupled model. The importance of coupling was explored by the following procedure.

We consider a joint whose response can be exactly predicted by the coupled joint model. We model the joint by the simple joint model and assess the importance of coupling by comparing the exact response to that of the identified uncoupled joint model

This procedure is implemented as follows. The structure with the coupled joint model is loaded, and measured displacements are simulated by the procedure described in Sec. V. Both the uncoupled and coupled models are identified from the simulated measurements and they are assembled with the remaining structure. Then, we apply the same loads (not necessarily the ones used for identification) to each structure and compare the resulting displacements and strain energies. The comparison process is shown in Fig. 8.

The two models predict different responses for the same loading. Among all possible loadings, there is one that corresponds to the largest difference between the strain energies of the two models. This loading case will be called maximum difference in the strain energies (MDSE) loading. MDSE loading is determined by maximizing the difference between the strain energies of the two models normalized by the square of magnitude of the forcing vector. Therefore, we maximize the following quantity,

$$\Delta E = \frac{E_{\text{complex}} - E_{\text{simple}}}{\boldsymbol{F}^T\boldsymbol{F}} \tag{12a}$$

E_{complex} and E_{simple} are given by

Table 4 Estimation results (car example)

Joint stiffness parameter	Measurements at joint only: Value[a]	Measurements at joint only: Standard deviation, percentage of actual value	Measurements away from joint: Value[a]	Measurements away from joint: Standard deviation (percentage of actual value)
k_{x12}	1.0644	2.50	1.1892	2.09
k_{y12}	1.1628	2.77	1.1712	2.42
k_{z12}	1.1423	3.69	1.1782	3.82
k_{x23}	1.2184	3.99	1.1596	3.33
k_{y23}	1.1663	3.16	1.1783	2.81
k_{z23}	1.1553	2.27	1.1365	2.07
k_{x31}	1.2624	6.42	1.1198	5.30
k_{y31}	1.1426	1.47	1.1407	1.30
k_{z31}	1.1423	4.28	1.1876	3.94

[a]The values of the parameters are given in 10^5 N·m.

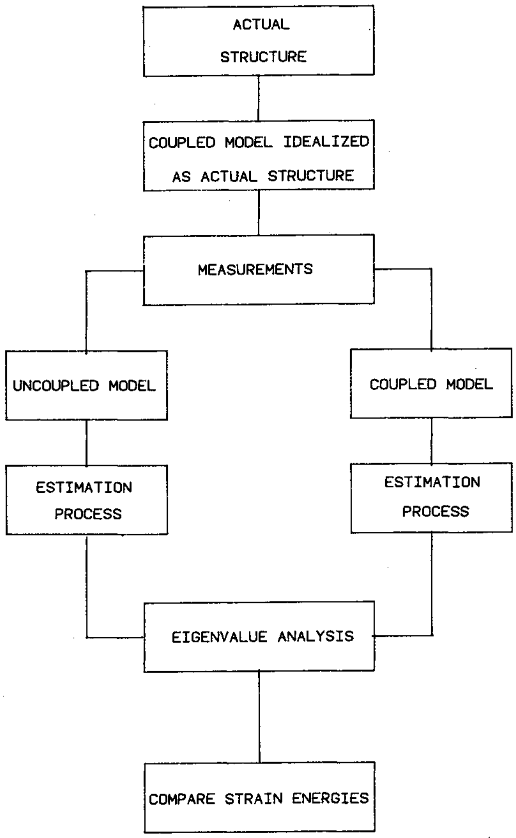

Fig. 8 Comparison process for coupled and uncoupled flexible joint models.

$$E_{\text{complex}} = \tfrac{1}{2}F^T K_{\text{complex}}^{-1} F \tag{12b}$$

$$E_{\text{simple}} = \tfrac{1}{2}F^T K_{\text{simple}}^{-1} F \tag{12c}$$

It can be shown that the load vector that maximizes ΔE is the eigenvector that corresponds to the largest eigenvalue of the matrix, which is the difference of the flexibility matrices of the complex and the simple model.[17]

The reduced stiffness matrix of the exact joint model is presented in Table 6. The stiffness matrices of the identified uncoupled and coupled models are also presented in the same table. We have used 20 leading cases to derive the results in Table 6. The estimates of the coupled model parameters were poor for fewer loading cases.

Next, we evaluate the MDSE loading corresponding to the two estimated models. One of the joint branches (branch 3) is clampled so that the joint becomes statically determinate. The MDSE loading practically constitutes of a unit moment in the θ_{x2} direction, which corresponds to the parameter k_{44}. This result should be expected because the largest entry in the matrix, which is the difference between the flexibility matrices of the two joint models, corresponds to degree of freedom θ_{x2}.

It was found that the strain energies stored in the coupled and the uncoupled models, when subjected to MDSE loading, were 0.3358×10^{-5} and 0.0808×10^{-5} N·m, respectively. This is a 76% difference, which indicates that the two models are widely different. The difference in the behavior of the two models can only be attributed to the coupling between the motions of the joint brances because the only difference between the two models is in accounting for such coupling. Thus, coupling is important in this example.

The two finite element models for the cube that incorporate the estimated uncoupled and coupled joint models are compared. The strain energies of the cube-complex joint assembly and cube-simple joint assembly subjected to the MDSE loading corresponding to these assemblies were found to be 0.6154×10^{-4} and 0.5928×10^{-4} N·m, respectively. The strain energy of a cube-joint assembly in which the joint is infinitely flexible when the assembly is subjected to MDSE loading was found to be 0.6743×10^{-4} N·m. The difference between the strain energies of the cube with the coupled and the uncoupled model is roughy 40% of the corresponding difference of the cube with the infinitely flexible joint and the coupled joint model. This indicates that the coupling is important.

Table 6 Comparison of joint stiffness parameters of uncoupled and coupled models estimated from the same set of measured displacements

Joint stiffness parameter	Exact	Estimated (coupled model)	Estimated (uncoupled model)
k_{11}	0.6098E+7	0.6093E+7	0.166E+7
k_{12}	−0.7255E+6	−0.7237E+6	0.0
k_{13}	−0.7805E+5	−0.7896E+5	0.0
k_{14}	0.2627E+5	0.3134E+5	−0.4847E+6
k_{15}	−0.1502E+6	−0.1431E+6	0.0
k_{16}	0.2830E+7	0.2812E+7	0.0
k_{22}	0.6098E+7	0.6240E+7	0.3281E+7
k_{23}	0.7805E+5	0.7428E+5	0.0
k_{24}	−0.2551E+5	−0.4054E+5	0.0
k_{25}	0.1374E+5	0.7228E+5	−0.7254E+6
k_{26}	−0.1502E+6	−0.1295E+6	0.0
k_{33}	0.3122E+6	0.3108E+6	0.4401E+6
k_{34}	−0.1530E+4	−0.9172E+3	0.0
k_{35}	−0.2551E+5	−0.2547E+5	0.0
k_{36}	0.2627E+5	0.1992E+5	−0.1941E+6
k_{44}	0.3122E+6	0.3009E+6	0.1328E+7
k_{45}	0.7805E+5	0.8608E+5	0.0
k_{46}	0.7805E+5	0.7609E+5	0.0
k_{55}	0.6098E+7	0.6006E+7	0.2553E+7
k_{56}	−0.7255E+6	−0.6980E+6	0.0
k_{66}	0.6098E+7	0.6032E+7	0.2063E+7

The values of the parameters are given in N·m

VIII. Conclusions

The following are the conclusions of this work.

1) For the example considered, the methodology for identification was found to be able to estimate model parameters in a small number of iterations.

2) Modeling error induces bias in the parameter estimates. This effect cannot be reduced by increasing the number of loading cases.

3) Probabilistic system identification is preferable to a deterministic method that estimates the parameters from a number of displacements equal to the number of parameters.

4) In the solution procedure, the combination of fixed-point iteration scheme and Newton's method yields a faster convergence than Newton's method alone.

5) Coupling is important. The examples in Sec. VI indicate that, if coupling is ignored when modeling the joint, the strain energy might not be calculated accurately.

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